Άσκηση 1(ΜΕΘΟΔΟΣ ΑΝΤΙΚΑΤΑΣΤΑΣΗΣ)

A) $\int \frac{dx}{x\ln x \ln(\ln x)} = I_1$

$E\acute{\alpha}\nu \quad \text{u=lnx} \Rightarrow \text{du=}\frac{1}{x}dx \Leftrightarrow dx = xdu$

$\acute{A}\rho\alpha \quad \mathrm{I}_1 = \int \frac{xdu}{xu\ln u} = \int \frac{du}{u\ln u}$

$E\acute{\alpha}\nu \quad \text{lnu=z} \Rightarrow \text{dz=}\frac{1}{u}du \Leftrightarrow du = udz$

$\acute{A}\rho\alpha \quad \mathrm{I}_1 = \int \frac{udz}{uz} = \int \frac{dz}{z} = \ln z + c$

$\mathrm{I}_1 = \ln(\ln u) + c = \ln(\ln(\ln x)) + c$

B) $\int \frac{\ln x}{x(1+\ln^2 x)}dx = I_2$

$E\acute{\alpha}\nu \quad \text{u=lnx} \Rightarrow \text{dx=xdu}$

$\acute{A}\rho\alpha \quad \mathrm{I}_2 = \int \frac{u}{x(1+u^2)}xdu = \int \frac{u}{1+u^2}du = \frac{1}{2}\int \frac{(2u)du}{1+u^2} = \frac{1}{2}\int \frac{(1+u^2)'}{(1+u^2)}du = \frac{1}{2}\ln(1+u^2) + c$

Γ) $\int \frac{dx}{\sqrt[6]{x^5}} = \int x^{-\frac{5}{6}}dx = \frac{x^{-\frac{5}{6}+1}}{-\frac{5}{6}+1} + c = \frac{x^{\frac{1}{6}}}{\frac{1}{6}} + c = 6x^{\frac{1}{6}} + c = 6\sqrt[6]{x} + c$

Άσκηση 2ΜΕΘΟΔΟΣ ΡΗΤΗΣ ΣΥΝΑΡΤΗΣΗΣ)

A) $\int (4x-1)e^{(2x^2-x+4)}dx = \int (4x-1)\frac{1}{4x-1}[e^{(2x^2-x+4)}]'dx = \int (e^{2x^2-x+4})'dx = e^{2x^2-x+4} + c$

B) $\int \frac{7}{(x-2)(x+5)}dx = I_3$

Αναλύω σε απλούστερα κλάσματα

$$\frac{1}{(x-2)(x+5)} = \frac{\mathrm{A}}{x-2} + \frac{\mathrm{B}}{x+5} = \frac{A(\mathrm{x}+5)+\mathrm{B}(\mathrm{x}-2)}{(x-2)(x+5)}$$

$$\Leftrightarrow 1 = \mathrm{A}(\mathrm{x}+5) + \mathrm{B}(\mathrm{x}-2) \Leftrightarrow$$

$1 = x(A+B) + 5A - 2B \Leftrightarrow$

$A + B = 0 \qquad A=-B \Leftrightarrow A=\frac{1}{7}$

$5A - 2B = 1 \qquad -7B=1 \Leftrightarrow B=-\frac{1}{7}$

$Άρα \quad \frac{1}{(x-2)(x+5)} = \frac{1}{7(x-2)} - \frac{1}{7(x+5)} \Leftrightarrow$

$I_3 = \frac{7}{7}\int \frac{1}{x-2}dx - \frac{7}{7}\int \frac{1}{x+5}dx \Leftrightarrow I_3 = \ln|x-2| + \ln|x+5| + c$

Γ) $\int 4^{\sqrt{2x+1}}dx = I_4$

$Εάν \quad u=\sqrt{2x+1} \quad τότε \quad du=\frac{1}{2u}(2x+1)'dx \Leftrightarrow du = \frac{1}{u}dx \Leftrightarrow dx = udu$

$Άρα \quad I_4 = \int 4^u udu = \int (\frac{1}{\ln 4}4^u)'udu \Rightarrow I_4 = \frac{1}{\ln 4}\int (4^u)'udu = \frac{1}{\ln 4}[4^u u - \int 4^u du]$

$= \frac{1}{\ln 4}[4^u u - \frac{1}{\ln 4}4^u] + c = \frac{1}{\ln 4}4^u u - \frac{1}{(\ln 4)^2}4^u + c$

Άσκηση 3(ΟΡΙΣΜΕΝΟ ΟΛΟΚΛΗΡΩΜΑ)

A) $\int_{-2}^{2}(e^{4^x} + e^x)dx = [\frac{1}{4}e^{4^x} + e^x]_{-2}^{2} = \frac{1}{4}e^8 + e^2 - (\frac{1}{4}e^{-8} + e^{-2})$

$= \frac{1}{4}e^8 + e^2 - \frac{e^{-8}}{4} - e^{-2} = \frac{e^8}{4} + e^2 - \frac{1}{4e^8} - \frac{1}{e^2}$

Β) $\int_{e}^{e^2} \frac{dx}{x(\ln x)^3} = I_5$

Εάν u=lnx τότε du=$\frac{1}{x}dx \Leftrightarrow dx = xdu$

Εάν x=e$^2 \Rightarrow u = \ln e^2 = 2$

x=e $\Rightarrow$ u = lne =1

$$I_5 = \int_1^2 \frac{xdu}{xu^3} = \int_1^2 \frac{du}{u^3} = \int_1^2 u^{-3}du = [\frac{u^{-2}}{-2}]_1^2 = [-\frac{1}{2u^2}]_1^2 = -\frac{1}{8} + \frac{1}{2} = -\frac{1}{8} + \frac{4}{8} = \frac{3}{8}$$

Γ) $\int_1^2 (\frac{2}{x^3} - \frac{4}{x^2} - \frac{3}{x})dx = \int_1^2 (2x^{-3} - \frac{4}{x^2} - \frac{3}{x})dx = [2\frac{x^{-2}}{-2} + \frac{4}{x} - 3\ln x]_1^2 = [-\frac{1}{x^2} + \frac{4}{x} - 3\ln x]_1^2$

$$= -\frac{1}{4} + 2 - 3\ln 2 - (-1 + 4 - 0) = -\frac{1}{4} + 2 - 3\ln 2 + 1 - 3 = -\frac{1}{4} - 3\ln 2$$

Δ) $\int_{-2}^{1} \frac{dx}{x+3} = \int_{-2}^{1} [\ln(x+3]'dx = [\ln(x+3)]_{-2}^{1} = \ln 4 - \ln 1 = \ln 4$

Ε) $\int_{-1}^{3} (3x^3 - 5x^2 + 2x)dx = [\frac{x^4}{4} - 5\frac{x^3}{3} + x^2]_{-1}^{3} =$

$$\frac{3^4}{4} - 5\frac{3^3}{3} + 9 - (\frac{1}{4} + \frac{5}{3} + 1) = \frac{81}{4} - 45 + 9 - \frac{1}{4} - \frac{5}{3} - 1 = \frac{80}{4} - 37 - \frac{5}{3} = -17 - \frac{5}{3} = -\frac{56}{3}$$

Άσκηση 4

Α) $\int e^{(2x+5)}dx = \int (\frac{1}{2}e^{(2x+5)})'dx = \frac{1}{2}e^{2x+5} + c$

Β) $\int \sigma\upsilon\nu 3x dx = \int (\frac{1}{3}\eta\mu 3x)'dx = \frac{1}{3}\eta\mu 3x + c$

Γ) $\int x^2 e^{2x^3} dx = \int x^2 (\frac{e^{2x^3}}{6x^2})'dx = \frac{1}{6}e^{2x^3} + c$

Δ) $\int \frac{xdx}{\sqrt{5x-3}} = I_6$

Εάν u=$\sqrt{5x-3}$ τότε du=$\frac{1}{2u}5dx \Leftrightarrow dx = \frac{2udu}{5}$

Άρα $I_6 = \int \frac{xdu2u}{5u} = \frac{1}{5}\int xdu$

Αλλά $u^2 = 5x-3 \Leftrightarrow x = \frac{u^2+3}{5}$

Άρα $I_6 = \frac{1}{25}\int (u^2+3)du = \frac{1}{25}(u^3+3u) + c$